

Algorithm for the computation of 3D Fourier descriptors

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Abstract

This work describes a new approach for the computation of 3D Fourier descriptors, which are used for characterization, classification, and recognition of 3D objects. The method starts with a polygonized surface which is mapped onto a unit sphere using an inflation algorithm, after which the polyhedron is expanded in spherical harmonic functions. Homogeneous distribution of the vertices is achieved by applying an iterative watershed algorithm to the surface graph.

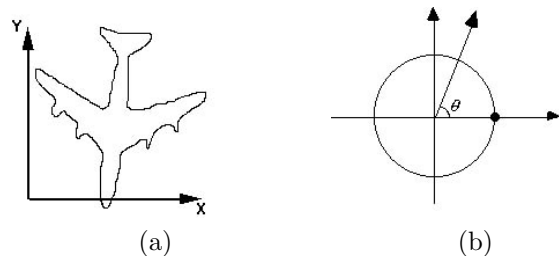


Figure 1. (a) Contour of a 2D object. (b): Unit circle as parameter space.

1 Introduction

Characterization or recognition of objects by means of Fourier descriptors (FDs) has extensively been described in the past, especially in two dimensions [10, 5, 4]. From the nineties on, FDs were also employed for 3D applications, first for 3D curves [3], later for 3D surfaces as well [6, 2, 8, 9]. FDs are used to characterize the objects contour in 2D or the objects surface in 3D. FDs are quite efficient for object parameterization because of properties such as transformation invariance, noise robustness, and uniqueness.

1.1 2D Fourier descriptors

For 2D object characterization, the procedure to obtain 2D FDs is well known [5, 4]. They are derived from the Fourier transform of the contour coordinates:

$$\vec{a}_n = \frac{1}{L} \int_0^L \vec{x}_l \exp\left(\frac{2\pi i n l}{L}\right) dl \quad (1)$$

with \vec{x}_l the coordinate vector of a point on a contour with length L . The Fourier coefficients \vec{a}_n of Eq. (1)

are then appropriately transformed to make them independent to translation, rotation, scale, and starting point. As can be seen from Eq. (1), the contour can adequately be parameterized by the angle $\theta = 2\pi l/L$, where the unit circle serves as parameter space (see Fig. 1). It is important to note that the sampling of the unit circle must be uniform. If this uniformity is not met, the orthogonality relation of the Fourier basis functions evaluated on discrete points on the unit circle, would not hold, which would result in erroneous Fourier descriptors. Fortunately, for a 2D contour, uniform subdivision of the unit circle along with a one-to-one mapping of the objects contour is trivial.

1.2 Extension to 3D

Where in 2D the contour is mapped onto a circle, in 3D the surface is mapped onto a sphere (see Fig. 2). This was already reported in the early nineties, where the surface of a (simple) 3D object was mapped to a sphere for posterior expansion in spherical harmonic

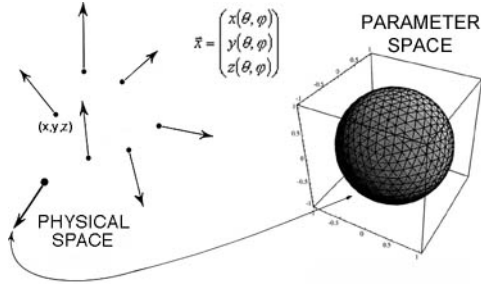


Figure 2. One-to-one mapping between the (x,y,z)-coordinates of the object to the angle parameters (θ, ϕ)

functions [6]. Later, Brechbühler, described a method to do this for objects that are more complex than convex or star-shaped (for these objects, the radial projection of the vertices onto the sphere is ambiguous)[2]. Compared to 2D, the mapping from the object space to the parameter space is far less obvious in 3D because the surface of a 3D object cannot be traced in an equally simple manner as can be done for the contour of a 2D region. During 3D-mapping, one has to pay attention to connectivity and uniformity:

Connectivity First neighboring nodes on the original object should also be first neighboring nodes on the sphere to ensure a unique one-to-one mapping. If not, spurious discontinuities are introduced in parameter space.

Uniformity Also, the object surface should be uniformly mapped onto the parameter space (i.e., the object points should be equally distributed onto the sphere). If not, higher frequencies and hence more Fourier Descriptors are needed to adequately describe the object.

The purpose of this paper is to propose a new method to obtain the uniform and unique mapping of the object's surface points to the parameter space.

2 Method

In this section, the various steps for the computation of 3D Fourier descriptors are described. Thereby, it is assumed that the surface of the object under concern is polygonized (i.e., described by vertices and polygons), and centered around the origin. Hence, without loss of generality, it is assumed that $\sum_{i=1}^N \vec{v}_i = \vec{0}$ with N the total number of vertices.

2.1 Mapping to a sphere

The first step is to map the polygonized surface onto a sphere. An interesting approach has been proposed by Brechbühler et al [1]. In their work, a continuous one-to-one mapping from the surface of the original object to the surface of a unit sphere was proposed. Thereby, two poles were selected on the surface, after which the vertices were uniformly mapped onto the sphere by means of a diffusion process. The disadvantage of this approach is that the iterative diffusion process is rather time-consuming, especially for a large number of vertices (> 5000).

In this section, we will describe a simple alternative approach for the mapping of a polygonized surface onto a sphere.

2.1.1 Inflation

The method we use to map the vertices of the original object to the sphere surface is an iterative blow-up algorithm in which the inflation of a crumpled balloon is simulated. The position of the center of the target sphere is chosen to be the center of the object, and the sphere's radius r_s is fixed at 2 times the maximum vertex radius of the object:

$$r_s = 2 \max_i |\vec{v}_i| \quad (2)$$

Each iteration consists of:

- **Moving towards a sphere** During each iteration, the vertices are non-linearly displaced towards the surface of a sphere. The rate with which a vertex \vec{v} proceeds depends on the distance of that vertex to the center of the target sphere. Vertices that are further away from the sphere surface proceed faster than vertices close to the sphere:

$$r_i^{new} = r_i^{old} + a \frac{|r_s - r_i^{old}|}{r_s} * (r_s - r_i^{old}) \quad (3)$$

with a a regulation constant.

- **Area equalization** During each iteration, the spatial coordinates of each vertex are adapted so as to equalize the areas of the surrounding polygons:

$$\vec{x} = \frac{\sum_{k=1}^K A_k \vec{c}_k}{\sum_{k=1}^K A_k} \quad (4)$$

where \vec{c}_k and A_k are the center and area of a polygon having vertex \vec{v}_k , respectively, where K denotes the number of surrounding polygons.

This procedure is iterated until the object becomes a star-shaped object, after which a final projection onto a unit sphere is performed.

2.1.2 Watershed transformation

The result of the previous phase is a mapping of the object's vertices onto the surface of a unit sphere. Thereby, topology is preserved, i.e., neighboring vertices on the original object are also neighboring vertices on the sphere surface. However, the polygons may not be distributed homogeneously over the sphere. As mentioned above, it is important for posterior computation of the Fourier Descriptors that the polygons are distributed as homogeneously as possible.

To obtain a homogeneous distribution, polygons with small areas should move towards parts on the sphere where the polygon-distribution is less dense. Moving one polygon, however, influences all neighboring polygons, because the polyhedron is in fact one large coupled system. Hence, global directions need to be found for each polygon to move to. This can be done by computing the watershed transform on the area graph of the spherical polyhedron. The procedure is summarized as follows:

- The polyhedron is transformed into a graph, where the value of each node of the graph corresponds to the area of a polygon.
- The local minima and maxima corresponding to each node are computed using the efficient immersion based watershed transformation proposed by Vincent and Soille [7]. Hence, for each polygon, we have the centers of those polygons that have locally maximal and minimal areas.
- The vertices of each polygon are then moved towards the local maximum and away from the local minimum polygon center, where the displacement vector is weighted by the area mean of the faces surrounding the extremum.
- Each vertex is then recomputed according to Eq. (4) to equalize the areas of the polygons it belongs to.

The above procedure is iterated until the difference between the maximum and minimum area is smaller than a certain threshold.

2.2 3D Fourier descriptors

The final step is the computation of the Fourier descriptors. Assume a spherical field $S(\vec{n})$, where $\vec{n} = (\cos \phi \cos \theta, \cos \phi \sin \theta, \sin \phi)$ is a vector, denoting a location on the unit sphere. In our application, S is a vector field denoting the spatial coordinates of a surface point on the original object. Furthermore, assume that there are N observations S_i available at locations

\vec{n}_i . Then, the field S at a new location \vec{n} can be approximated from the observations using spherical harmonic functions Y_l^m :

$$\hat{S}(\vec{n}) = \mathbf{a} \cdot \mathbf{y}(\vec{n}) = \sum_{l=0}^L \sum_{m=-l}^l a_l^m Y_l^m(\vec{n}) \quad (5)$$

where the Fourier coefficients a_l^m are the inner product of the field S with the spherical harmonics Y :

$$a_l^m = \int_0^\pi \int_0^{2\pi} S(\theta, \phi) Y_l^m(\theta, \phi) d\phi \sin \theta d\theta \quad (6)$$

In practical applications, these coefficients are found, for example, by least squares estimation of the spherical harmonics from the discrete observations:

$$\mathbf{a} = \sum_{i=1}^N S(\vec{n}_i) \mathbf{y}(\vec{n}_i)^H (\mathbf{Y}^H \mathbf{Y})^{-1} \quad (7)$$

where $\mathbf{y}(\vec{n}) = [Y_0^0(\vec{n}), Y_1^{-1}(\vec{n}), Y_1^0(\vec{n}), \dots, Y_L^L(\vec{n})]^T$ and $\mathbf{Y} = [\mathbf{y}(\vec{n}_1), \dots, \mathbf{y}(\vec{n}_N)]^T$ and H denotes the Hermitian transpose. Note that the above equation simplifies significantly in case the orthogonality relation

$$\int_0^\pi \int_{-1}^1 Y_{l'}^{m'}(\theta, \phi) Y_l^m(\theta, \phi) d\phi \sin \theta d\theta = \delta_{l',l} \delta_{m',m} \quad (8)$$

holds (i.e., for regularly spaced observations). Finally, the coefficients of Eq. (6) are made independent with respect to translation, rotation, and scale to obtain the Fourier descriptors [1].

3 Results and discussion

Fourier descriptors from various objects were computed using the above method. As an example, a three-dimensional 'E'-object is shown in Fig. (3a), which is non-convex. After polygonization, the object was mapped onto a unit sphere using the proposed inflation algorithm. Several steps of this phase are shown in Fig. (3).

One can easily observe that the vertices of the original object are mapped onto the unit sphere while connectivity is preserved. The number of iterations required for the mapping, strongly depends on the object form. Convex objects in principle do not need this preprocessing step as direct radial projection of the vertices onto the sphere would correctly map the mesh onto the sphere surface. Non-convex objects, however, require the inflation step. For the E-object, for example, about 100 iterations were required for the mapping. Another 100 iterations were used to make the polygon

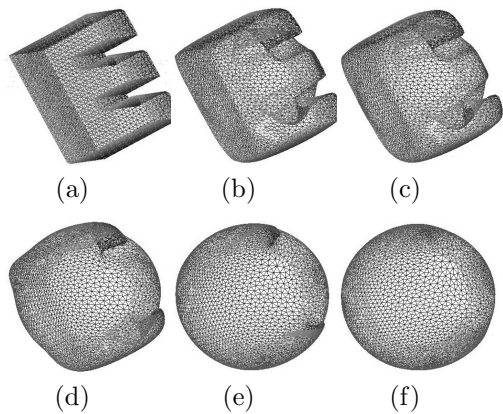


Figure 3. Inflation-algorithm. (a) Original object ; (b) $N = 1$; (c) $N = 2$; (d) $N = 12$; (e) $N = 30$; (f) $N = 100$ with N the number of iterations.

distribution uniform. The results for this object are summarized in Table 1. Finally, Fig. 4 shows two reconstructions of the E-object: the first and the eighth order reconstruction.

| # vertices | # iterations | time (s) |
|------------|--------------|----------|
| 5000 | 100 | 26 |
| 20000 | 100 | 112 |
| 50000 | 100 | 306 |

Table 1. Experimental results for the E-object

3.1 Limitations of the method

The proposed method is generally applicable to various object forms, which may be non-star-shaped or non-convex. However, objects containing holes, or of which the spatial mean of the vertices does not lie within the object (i.e., in the background) are not suited for this mapping as the inflation phase would not work. The solution for this might be to choose one or more points within the object from which the inflation is started.

4 Conclusions

In this paper, a new approach was discussed to map a polygonized 3D object to a unit sphere, which is a necessary step for the computation of 3D Fourier descriptors. An inflation algorithm was proposed which

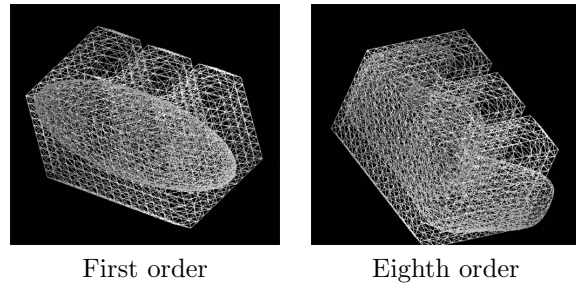


Figure 4. First and eighth order reconstruction of the E-object.

efficiently maps the vertices of the original object to the sphere. A uniform distribution was obtained using the watershed transform.

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