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● *Technical Note*

ESTIMATION OF THE NOISE IN MAGNITUDE MR IMAGES

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Magnitude magnetic resonance data are Rician distributed. In this note a new method is proposed to estimate the image noise variance for this type of data distribution. The method is based on a double image acquisition, thereby exploiting the knowledge of the Rice distribution moments. © 1998 Elsevier Science Inc.

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INTRODUCTION

Estimation of the image noise variance (NV) is important for several reasons. Firstly, it provides a measure of the image quality in terms of image detail. Furthermore, knowledge of the NV is useful in the analysis of the magnetic resonance (MR) system: e.g., to test the performance of the MR system itself (receiver coil, preamplifier, etc.). Also, the NV is an important quality measure in functional MR imaging, where signal variations of the order of a few percent need to be detected. Finally, the NV value is often used as input for image processing techniques such as image restoration[1,2] or image filtering.[3,4]

Commonly, the image NV is estimated from a single magnitude image; thereby the NV is determined directly from a large uniform signal region or from non-signal regions.[5,6] Although these methods may lead to useful NV estimates, large homogeneous regions are often hard to find, such that only a small amount of data points are available for estimation. Also, back-ground data points sometimes suffer from systematic intensity variations. To cover these disadvantages, methods were developed based on two acquisitions of the same image: the so-called double acquisition methods. Thereby, the amount of noise is for example computed by subtracting two acquisitions of the same object and calculating the standard deviation of the resulting image pixels.[7] Murphy et al.[8] elaborated this technique further and used a parallel rod test object for NV measurements from the signal and non-signal blocks. The double acquisition methods have

an advantage over the single image techniques in that they are relatively insensitive to structured noise such as ghosting, ringing and direct current artefacts. However, a strict requirement is the perfect geometrical registration of the images. To overcome this restriction, recently a cross-correlation technique of the two acquisitions was suggested.[9] Besides geometrical registration, another problem may arise: due to small timing errors the raw data from one acquisition may be shifted relative to the other. After Fourier transformation, this results in different phase variations of the complex data such that the above double acquisition NV estimation methods are no longer valid. To overcome this problem, we propose an NV estimation method based on two magnitude MR images.

In magnetic resonance imaging (MRI), the acquired complex data is known to be corrupted by white noise having a Gaussian probability distribution (PD). After inverse Fourier transformation the real and imaginary data is still corrupted with Gaussian noise because of the orthogonality of the Fourier transform. Although all information is present in the real and imaginary images, it is common practice to work with magnitude and phase images instead as they have more physical meaning (proton density, flow, etc.). However, computation of a magnitude image is a non-linear operation which changes the data distribution. It can be shown that the data in a magnitude image is no longer Gaussian but Rician distributed.[5,10] In this note it is demonstrated how

the properties of this distribution can be exploited to estimate the image noise variance.

This paper is organized as follows. We first review briefly the properties of the Rice distribution. Then we show how these properties can be exploited to estimate the image noise variance using a double image acquisition. The proposed method is first tested on an artificial image. This was done because in a controlled situation, unforeseen errors, such as a bias, can be detected. Finally, the method is tested on various MR images.

THE RICE DISTRIBUTION

If the real and imaginary data, with mean values A_R and A_I respectively, are corrupted by zero mean Gaussian, stationary noise with standard deviation σ, it is easy to show that the PDF of the magnitude data will be a Rician distribution (pages 138–139 of[11]), given by:

$$p_M(M \mid A) = \frac{M}{\sigma^2} e^{-\frac{M^2 + A^2}{2\sigma^2}} I_0\left(\frac{AM}{\sigma^2}\right) \qquad (1)$$

where I_0 is the zero order modified Bessel function of the first kind. M denotes the pixel variable of the magnitude image and A is given by:

$$A = \sqrt{A_R^2 + A_I^2} \qquad (2)$$

Notice that the Rice distribution tends to a Rayleigh distribution when the signal-to-noise ratio goes to zero (i.e., when $A/\sigma \rightarrow 0$) and approaches a Gaussian distribution at high signal-to-noise ratio (i.e., when $A/\sigma \rightarrow \infty$).

Moments of the Rice Distribution

The νth moment of the Rice density function is given by:

$$E[M^\nu] = \int_0^\infty \frac{M^{\nu+1}}{\sigma^2} e^{-\frac{M^2+A^2}{2\sigma^2}} I_0\left(\frac{AM}{\sigma^2}\right) dM \qquad (3)$$

The previous equation can be analytically expressed as a function of the confluent hypergeometric function $_1F_1$:

$$E[M^\nu] = (2\sigma^2)^{\nu/2} {_1F_1}\left[-\frac{\nu}{2}; 1; -\frac{A^2}{2\sigma^2}\right] \qquad (4)$$

For some particular moments, i.e., when $\frac{\nu}{2}$ is an integer, the confluent hypergeometric function becomes a simple polynomial in its argument. Particularly the second moment is given by:

$$E[M^2] = 2\sigma^2 + A^2 \qquad (5)$$

NOISE VARIANCE ESTIMATION

From Eq. (5) one can determine the noise variance σ^2. In MR imaging, a common way to unbiasedly estimate σ^2 of a magnitude image is by estimating $E[M^2]$ from a spatial average of the squared background data points, where A is known to be zero:[6,10,12,13]

$$\widehat{\sigma^2} = \frac{1}{2}\langle M^2 \rangle \qquad (6)$$

This approach usually requires user interaction to select the background pixels.

We propose an alternative noise estimation scheme using a double acquisition scheme. When two images are acquired under identical imaging conditions, one can solve σ^2 from two equations and two unknowns using the averaged and single images, since:

$$E[\langle M_s^2 \rangle] = \frac{1}{N}\sum_{n=1}^N A_n^2 + 2\sigma^2 \qquad (7)$$

$$E[\langle M_a^2 \rangle] = \frac{1}{N}\sum_{n=1}^N A_n^2 + 2\left(\frac{\sigma}{\sqrt{2}}\right)^2 \qquad (8)$$

where $\langle \rangle$ denotes a spatial average of the whole image. The subscripts s and a refer to the single and averaged images, respectively. From Eq. (8) and Eq. (7) an unbiased estimator of the noise variance is derived:

$$\widehat{\sigma^2} = \langle M_s^2 \rangle - \langle M_a^2 \rangle \qquad (9)$$

This approach has the following advantages:

- it does not require any user interaction as no background pixels need to be selected;
- it is insensitive to systematic errors as long as these appear in both images. It is clear that, if this type of error appears in only one of the two images, none of the double acquisition methods yields the correct result;
- the precision of the noise variance estimator is drastically increased as all the data points are involved in the estimation;
- it is valid for any image signal-to-noise ratio.

An obvious disadvantage is the double acquisition itself. However, in MR acquisition schemes it is common practice to acquire two or more images for averaging. Hence, those images may as well be used for the proposed noise estimation procedure without additional acquisition time.
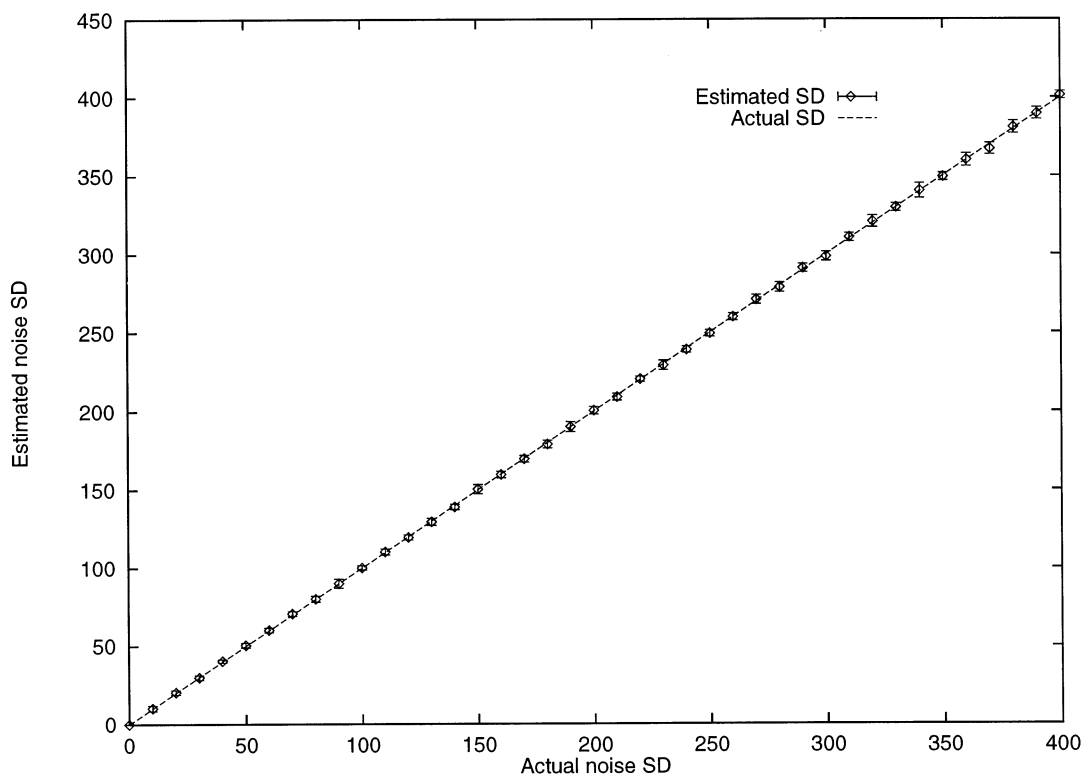
Fig. 1. Estimation of the noise variance: simulation experiment.

EXPERIMENTS AND DISCUSSION

The performance of the noise variance estimation method was in a first phase tested on an artificial image: the ''Lena'' image, well known in image processing. This was done because it is a controlled situation in which a possible bias should be revealed if present. The dimensions of the image were 128×128. From the Lena image, two independent Rician distributed images with standard deviation σ were generated. The proposed noise estimation method was tested for various values of σ. Simulation results (see Fig. 2) show a perfect linear behavior with unit slope of the estimated NV as a function of the true NV, demonstrating the accuracy of the method. Futhermore, the NV estimation was observed to be highly precise owing to the fact that all image pixels were used in the estimation.

Next to the test images, the noise estimation method was applied to magnetic resonance images. The data were generated using an MR apparatus (SMIS, Surrey, England) with a horizontal bore of 8 cm, a main magnetic field strength of 7 Tesla and a maximal gradient strength of 0.1 Tesla/m. In all experiments a birdcage radiofrequency coil with a diameter of 32 mm was used. The method was tested using two-dimensional spin echo as well as gradient echo sequences. The object imaged was initially a vegetable (cucumber) and secondly an animal (head of a mouse). For each experiment 20 images of size 256×128 were acquired with repetition time = 500 ms and echo time = 30 ms. For each object, the number of averages was varied from 2 until 32 with step 2. For each NA, 20 images were acquired, allowing 10 independent noise estimations from two images using Eqs. (7–8). Only the mean value and standard deviation of the 10 NV estimates were retained.

No artefacts were observed in the images of the cucumber though small ringing and ghosting artefacts were present in the images of the mouse head. These, however, did not influence the NV estimations as they appeared in both images. The results of the experiment (for the mouse head) are shown in Fig. 1 where the inverse NV estimates are plotted as a function of the number of averages, along with their 95% confidence intervals. As can be observed from the figure, the results are in correspondence with the theory in that the inverse NV estimates increase linearly with the number of averages, with zero offset.
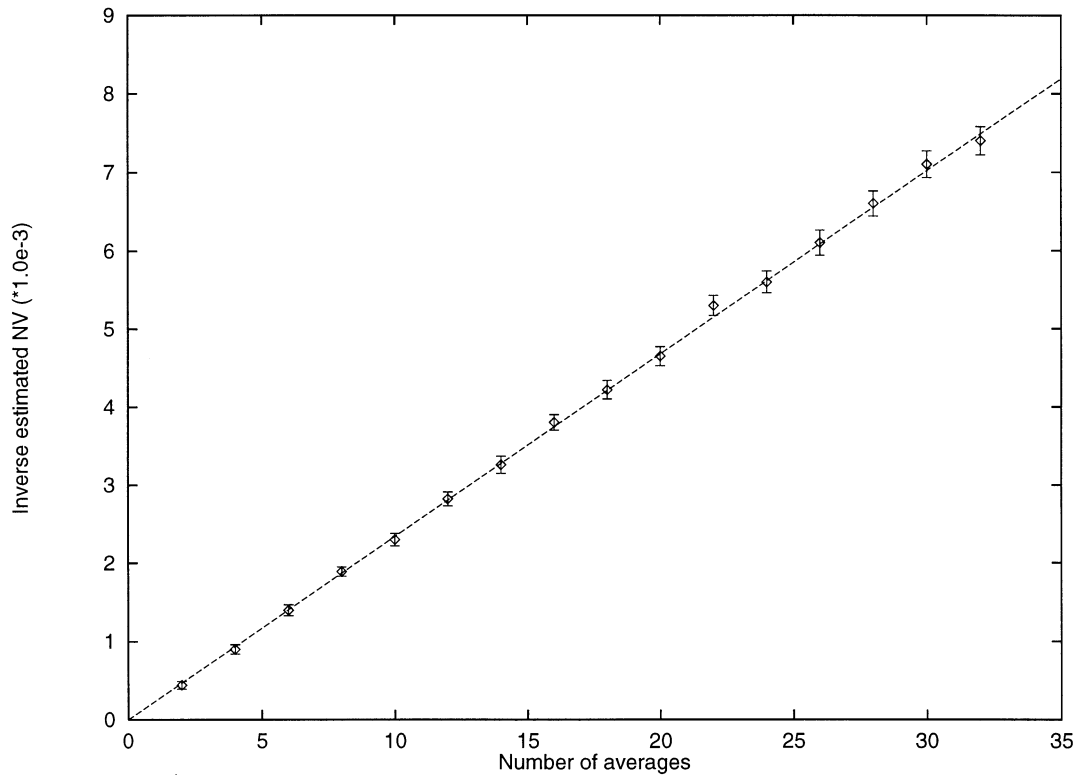
Fig. 2. Estimation of the noise variance from magnitude MR images.

CONCLUSION

When it comes to estimation of the image noise variance, methods based on a double acquisition are far superior to single image techniques in terms of precision. However, existing double acquisition methods become useless when different phase variations are present in the two images. To overcome this problem, a noise variance estimation method has been proposed based on two magnitude images. Under the condition of geometrical registration, the proposed noise variance estimator has been shown to be highly precise and accurate.

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