Projection-based polygon estimation in X-ray computed tomography

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Abstract

In X-ray computed tomography, the 3D structure of a scanned object can be reconstructed from a number of projection images of the object acquired from different directions. Conventional tomographic reconstruction algorithms represent the reconstructed volume on a voxel grid. Such representation is, however, not well suited for polyhedral objects arising in many industrial applications, since such objects are first voxelized during the reconstruction and then processed in order to obtain a polygon mesh representing the surface of the object. These transformations lead to loss of details and may induce artefacts that hinder posterior image processing.

In this work, a new approach is proposed in which a contour polygon of the object is directly estimated from the projection data. The approach is based on simulated projections of the polygon model and optimization of the vertex positions in the model with respect to the distance between the simulated and the original projection data (projection distance). The obtained results demonstrate the ability of the proposed algorithm to accurately represent the contour of the object even in case of noisy projection data.

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Introduction

X-ray computed tomography is an imaging technique that is capable of revealing the 3D structure of a scanned object with numerous applications in medicine, research (e. g., materials research), and industry (e. g., quality control). A typical workflow in industrial quality control involving computed tomography includes manufacturing of an object according to a CAD model, the reconstruction of the object using a tomographic reconstruction algorithm, segmentation, estimation of a polygon mesh representing the surface of the segmented object, and comparison with the CAD model used during manufacturing [1,2].

Conventional tomographic reconstruction algorithms, such as FBP [3], SIRT [4], or DART [5], represent the reconstructed volume on a voxel grid. This representation is not well suited for polyhedral objects arising in many industrial applications, since such representation can lead to loss of details in the object and to introduction of image artefacts [6]. Moreover, artefacts in the reconstructed volume can hinder posterior processing [1].

To generate a polygon mesh of the surface, contouring techniques, such as Marching Cubes [7] and its modifications, are widely used. These techniques create a triangular mesh using interpolation between attenuation values calculated on the voxel grid. Recently, a technique [8] has been proposed, where reconstruction on an iteratively deformed tetrahedral mesh is used instead of reconstruction on a regular voxel grid. These methods, however, are sensitive to noise and reconstruction artefacts and require a high resolution reconstruction to reproduce sharp edges on an object [7,8].

In this work, a new approach is proposed in which parameters of a model representing the object are directly estimated from the projection data, eliminating need for reconstruction on the pixel (voxel) grid and therefore decreasing related drawbacks. Our approach is based on optimization of an analytical model of the object with respect to a difference between the simulated projections of the model and the original projection data. Such models are readily available in many application domains, e. g., in industrial quality control, where objects being controlled are manufactured according to CAD models. In the present paper, we focus on the two-dimensional case and use polygons as the models to estimate the contour of the object. Two particular problems, *Edge estimation* and *Polygon estimation*, relevant for diamond processing, are considered.

Method

In tomography, the projection process can be modeled as a linear operator determined by the projection geometry, which leads to a system of linear equations

$$Wx = p\,, \tag{1.1}$$

where $p \in R^m$ is the projection data, $x \in R^n$ is an image representing the object on a pixel (voxel) grid, $W \in R^{m \times n}$ is the projection matrix with m being the number of detector elements multiplied by the number of projection angels and n being the number of pixels in the image. For any image $s \in R^n$ we can define the difference between its simulated projections and the measured projection data as

$$d(s) = \|Ws - p\|_2, \qquad (1.2)$$

which is known as the *projection distance*. Analogously, a projection operator W_M for a model-based representation of the object and the projection distance $d_M(t) = \|W_M t - p\|_2$ for any model t can be defined.

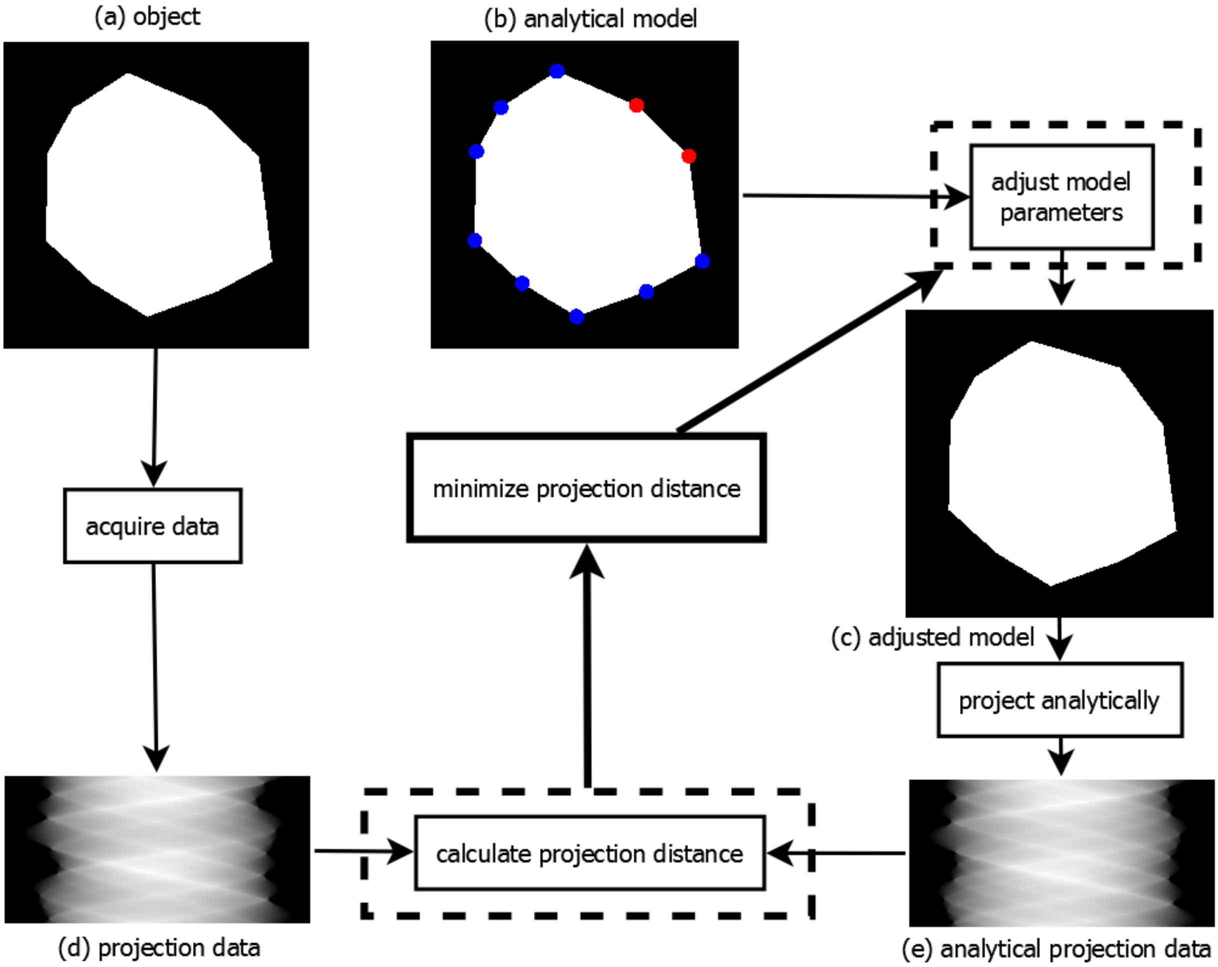
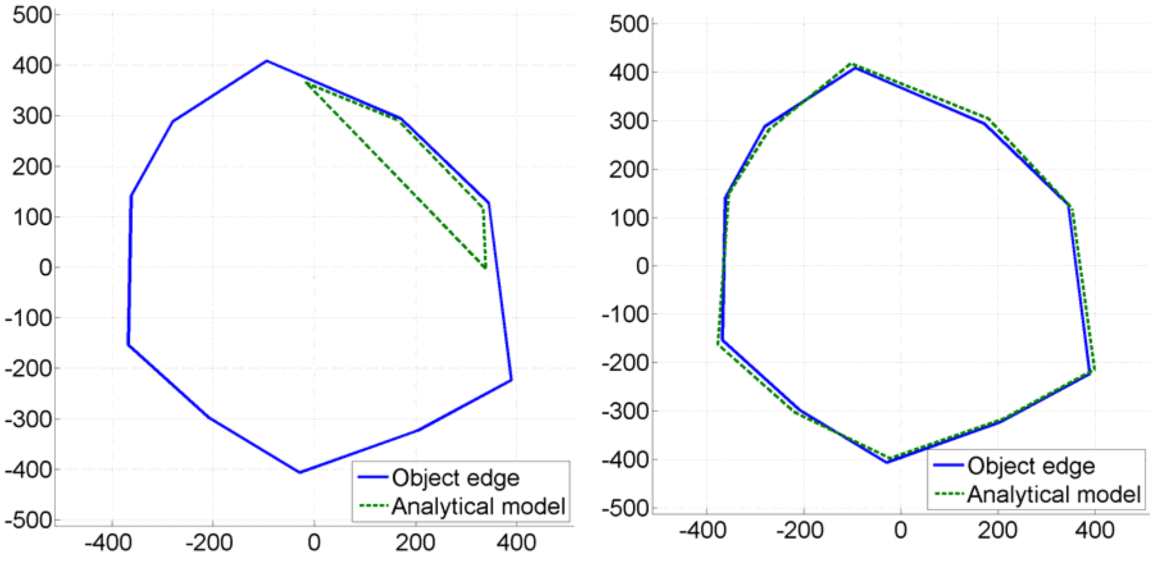


Figure 1: *Schematic overview of the proposed model estimation approach.*

Our approach is to find the parameters of the model t of the object that minimize the projection distance $d_M(t)$ between the acquired projection data and simulated projections of that model.

Consider projection data p (Figure 1d) which was acquired from the unknown object (Figure 1a). The approach consists in adjusting parameters of a model of the object (Figure 1b), which is then analytically projected. Next, the obtained projection data of the model (Figure 1e) is compared to the measured projection data of the object using the projection distance. The parameters that minimize the projection distance are retained as the parameters of the model representing the object.

The above described approach is very general and can be applied to a variety of models representing objects arising in different domains where computed tomography is applied. In the present paper, we consider two particular problems relevant for diamond processing.



a) b)

Figure 2: *(a) Edge estimation and (b) Polygon estimation.*

In *Edge estimation* (Figure 2a), we aim at estimating the slope of one edge of the object assuming that an approximate position and slope of that edge is available and there is no prior model of the object. A trapezium is used as an analytical model, parameterized with rotation, shift, length of the top basis and the base angles. The optimization of the projection distance is performed only for a part of the available projection data that is located in proximity to the projections of the edge in question and corresponds to X-rays being roughly parallel to it.

In *Polygon estimation* (Figure 2b), a complete object is estimated assuming the availability of a prior polygonal model representing the object. The optimization is iterative, and during each iteration each vertex is adjusted while keeping the other vertices fixed. This procedure allows to split the optimization in a high-dimensional search space into several optimizations in two-dimensional search spaces.

In both cases, we assume that the objects are homogeneous without any holes and that their attenuations are known. To optimize the projection distance, an interior-point method for nonlinear programming [9] is used. It is an iterative algorithm that combines a line search method computing steps by factoring the primal-dual equations and a trust region method employing conjugate gradient iterations. Bounds for the parameters of the model can be used to restrict the search space. These bounds depend, in principle, on the accuracy of the prior model.

Experiments

Simulation experiments

A number of experiments were set up by simulating projection data of a polygonal phantom to demonstrate the proposed approach. In all experiments described in this section, a detector with 1044 elements was used, 500 projections were computed and Poisson noise was added to obtain 5 datasets for each noise level.

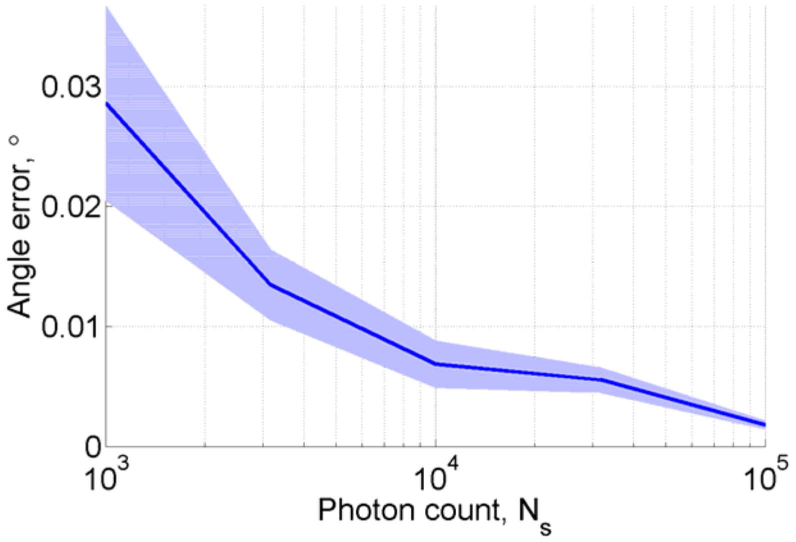


Figure 3: *Angle error as a function of the photon count for the edge estimation.*

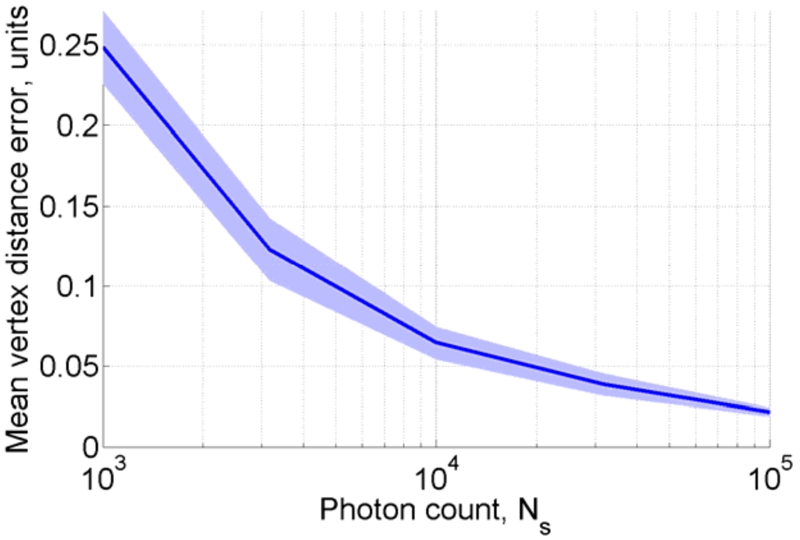


Figure 4: *Mean vertex shift error as a function of the photon count for the polygon estimation.*

In the first series of experiments, the ability of the proposed approach to handle Edge estimation problem was evaluated. For each noisy dataset computed for the object shown in Figure 2a, the slope of one edge was estimated as described in Section Method, and the mean angle error was plotted (Figure 3) together with the standard error (shown as shaded area in the plot).

In the second series of experiments, the polygon representing the object was estimated using the prior model (shown in Figure 2b together with the object), which was obtained from the object by randomly shifting each vertex, the mean shift was 12.17 units. For each noisy dataset the vertex positions were estimated, and the mean distances between the true and estimated vertex positions were calculated (Figure 4).

The results demonstrate the ability of the proposed approach to achieve subpixel accuracy in polygon estimation (Figure 4) and to accurately determine edge slopes (Figure 3), confirming that the proposed approach can accurately estimate parameters of a model representing the object based on the projection data even in the presence of noise.

Real experiments

Two cone-beam datasets were acquired using a desktop micro-CT system SkyScan-1172 (Bruker-MicroCT, Belgium), size of each projection image was 1000×524. The objects were known to be homogeneous, their attenuation was calculated as the mean attenuation in an inner region. One slice from each dataset was used as the projection data to demonstrate the performance of the proposed approach on real data.

For the first experiment, 280 projections of a diamond with a polished facet were acquired. The proposed approach was applied to estimate the slope of the polished facet, for which an initial model was roughly determined using SIRT reconstruction of the same slice (Figure 5a). The obtained result is shown in Figure 5b.

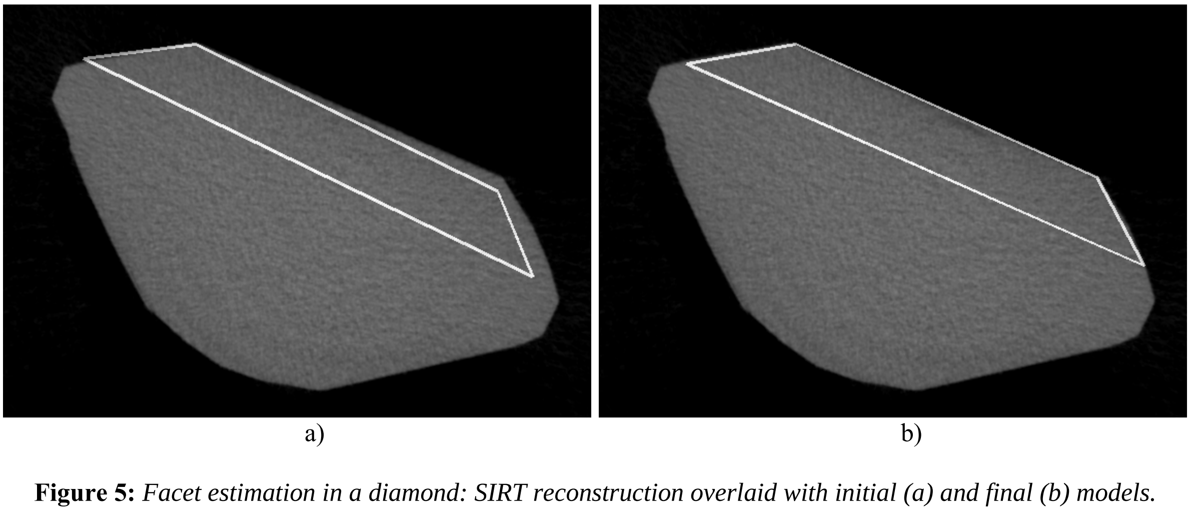
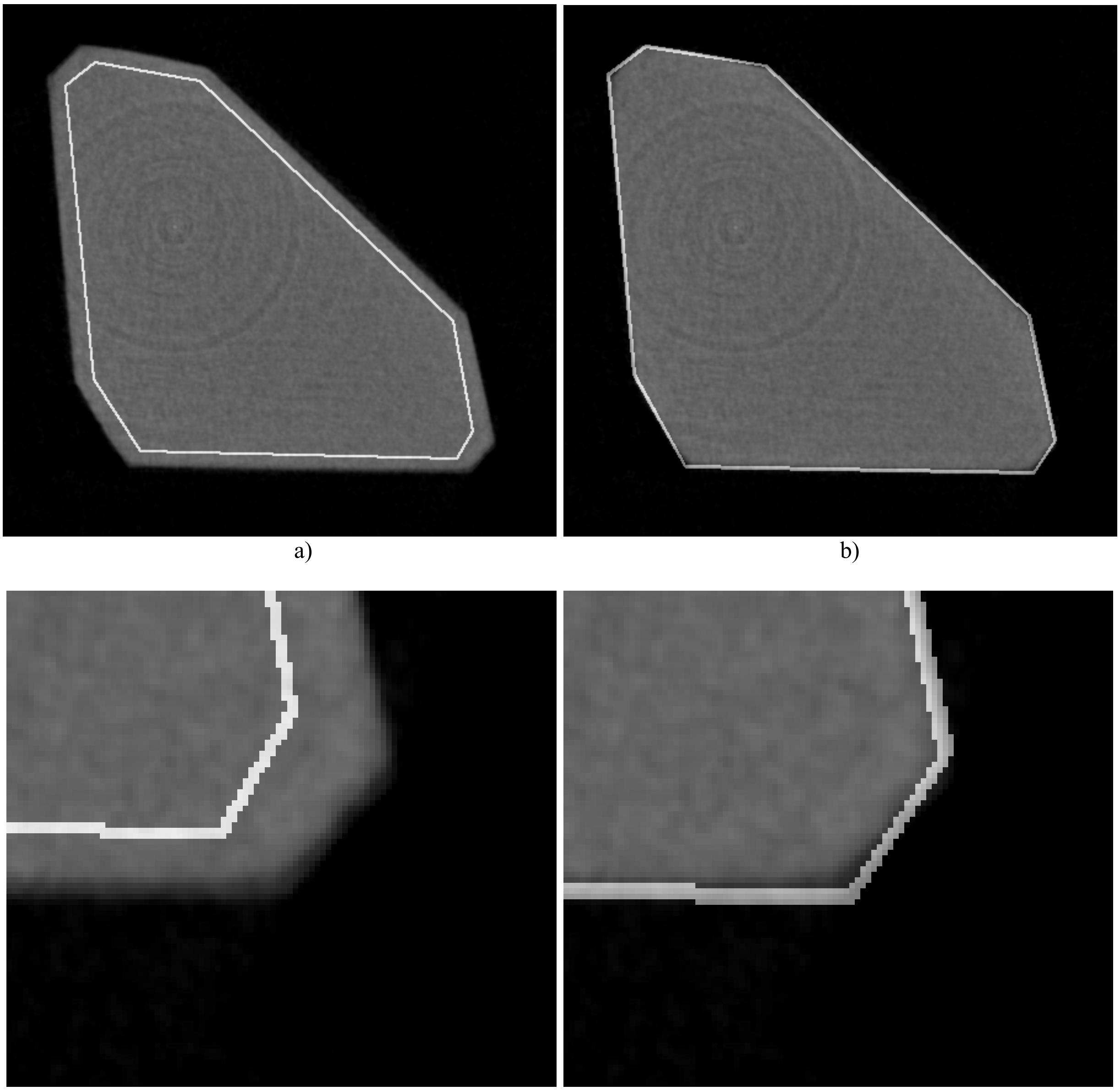

a) b)

Figure 5: *Facet estimation in a diamond: SIRT reconstruction overlaid with initial (a) and final (b) models.*

In the second experiment, 515 projections of an almost polished diamond were acquired. The polygon representing an initial model was again roughly determined using SIRT reconstruction (Figure 6a) and supplied into the proposed approach. The resulting polygon is presented in Figure 6b. While the final model may appear to slightly overestimate the object, e. g., in the bottom right corner of the figure, enlarged plots (Figures 6c, 6d) suggest the presence of material there, and the lower attenuation of the object in that area might have been caused by a reconstruction artefact.



a) b)

c) d)

Figure 6: *Polygon estimation in a diamond: SIRT reconstruction overlaid with initial (a) and final (b) models and the corresponding enlarged fragments (c) and (d).*

The proposed model estimation approach demonstrates plausible and promising results on the real datasets, allowing to compute the parameters of the models representing the scanned objects, and is potentially robust against reconstruction artefacts. Nevertheless, further validation of this approach is required.

Discussion and conclusion

In this paper, a projection-based polygon estimation approach for X-ray computed tomography was proposed. In this approach, a model representing the contour of the object is directly estimated from projection data, eliminating need for reconstruction on the pixel (voxel) grid. While polygon models were used to represent the objects throughout the paper, this technique is readily extensible to other representations, such as splines.

Experiments show that the proposed approach can accurately estimate the parameters of the models representing the objects even in the presence of noise and has great potential to enhance diamond processing or quality assessment of industrial parts.

Future work will focus on the extension of the proposed approach to three-dimensional models and non-homogeneous objects and further validation on real datasets.

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