Joint reconstruction and flat-field estimation using support estimation

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I. Introduction

X-RAY computed tomography reconstructions frequently suffer from ring-artefacts. Caused by the non-uniformity of the detector, these artefacts appear as light or dark rings in the reconstruction [1]. Currently, variations in the response of detector elements is estimated by the use of flat-fields, scans without an object in the scanner. Any discrepancies between the flat-fields and the measured data will lead to ring artefacts in the reconstruction, as the flat-fields are used to normalize each projection [2].

Most methods for the reduction of ring artefacts are either pre-processing methods where the non-uniformity is reduced in the projection data [3] or post-processing methods where rings are removed from the reconstruction [4]. In contrast to the processing methods, both Aggrawal [2] and Paleo [5] have suggested algorithms for estimating flat-fields during reconstruction. In both papers a non-linear minimization with a Total Variation prior is described.

We propose a joint linear system of equations which results in both a reconstruction and a flat-field estimate. Our proposed model is mathematically simpler and can be solved by fast linear Krylov methods. Furthermore, the proposed method uses no prior knowledge and can therefore be incorporated in any existing workflow.

II. Method

The acquired projection data from a CT system can be described using the Beer-Lambert law, which states that the projection of an object with a spatial attenuation function $\mu(\boldsymbol{x})$ under an angle θ and a signed distance s from the origin, can be described as

$$\ln(I(s,\theta)) = \ln(I_0(s)) - \int \mu(\boldsymbol{x})\delta(\boldsymbol{x}\cdot\boldsymbol{\xi} - \boldsymbol{s})\,\mathrm{d}\boldsymbol{x} \qquad (1)$$

with I the measured intensity, I_0 the incoming intensity and $\boldsymbol{\xi} = \begin{bmatrix}\cos(\theta) & \sin(\theta)\end{bmatrix}^\top$ [6]. The integral equation (1) can be discretised as follows:

$$b_{m,k} = z_k - \sum_j a^m_{k,j} y_j \qquad (2)$$

where the index k denotes a specific ray from the source to detector element k, the index m denotes the projection, $\boldsymbol{b} = b_{m,k} = \ln(I(m,k)) \in \mathbb{R}^K$, K the total number of detector elements, $\boldsymbol{z} = (z_k) = \ln(I_0(k)) \in \mathbb{R}^K$, $\boldsymbol{y} = (y_n) \in \mathbb{R}^N$ a discretised version of μ, N the total number of voxels of the reconstruction and $\boldsymbol{A}_m \in \mathbb{R}^{K\times N}$ the system matrix for the m'th projection. We assume the detector response to vary over the detector pixels

$$\boldsymbol{b}_m = \boldsymbol{I}\boldsymbol{z} - \boldsymbol{A}_m\boldsymbol{y}, \qquad (3)$$

with $\boldsymbol{I} \in \mathbb{R}^{K\times K}$ a unit matrix. Further assuming that the detector response is independent of the projection angle leads to the following linear system for M total angles:

$$\boldsymbol{b} = \boldsymbol{F}\begin{bmatrix}\boldsymbol{y}\\ \boldsymbol{z}\end{bmatrix}, \quad \boldsymbol{F} = \begin{bmatrix}-\boldsymbol{A}_1 & \boldsymbol{I}\\ \vdots & \vdots\\ -\boldsymbol{A}_M & \boldsymbol{I}\end{bmatrix}. \qquad (4)$$

The least squares solution to this linear system of equations can be found by solving the equivalent system

$$\boldsymbol{F}^\top\boldsymbol{F}\begin{bmatrix}\boldsymbol{y}\\ \boldsymbol{z}\end{bmatrix} = \boldsymbol{F}^\top\boldsymbol{b}. \qquad (5)$$

We will solve this system with a fast Krylov subspace method called Conjugate Gradients (CG).

The column space of the system matrix $\boldsymbol{A}$ is linearly dependent on the flat-field operator. This can be easily seen as for example the projection of a single voxel in the rotation center is a straight line, as its projection is independent of the angle, so it is indistinguishable from a non-uniformity in detector response in the center of the detector. We propose to reduce the dimension of the solution space by first determining a rough estimate of the support of the reconstruction. This can be done by thresholding a preliminary reconstruction obtained after a very low amount of CG iterations, typically less than five. This gives us an estimate S of the support of the object in the reconstruction domain. Using this support results in solving the system

$$-\boldsymbol{A}\boldsymbol{y} + \boldsymbol{U}\boldsymbol{z} = \boldsymbol{b}, \quad \boldsymbol{A} = \begin{bmatrix}\boldsymbol{A}_1\\ \vdots\\ \boldsymbol{A}_M\end{bmatrix}, \boldsymbol{U} = \begin{bmatrix}\boldsymbol{I}\\ \vdots\\ \boldsymbol{I}\end{bmatrix} \qquad (6)$$

subject to $y_s = \gamma$ for $s \notin S$, with S the support of the object and γ a fixed background value. The reconstruction and flat-field estimation get better with better estimations of the support. Fixing the voxels that do not belong to S is equivalent to deleting the columns of the system matrix $\boldsymbol{A}$ that correspond to these voxels. This reduces the amount of flat-fields in the column space of A and consequently improves the flat-field estimation.

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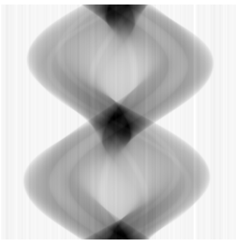
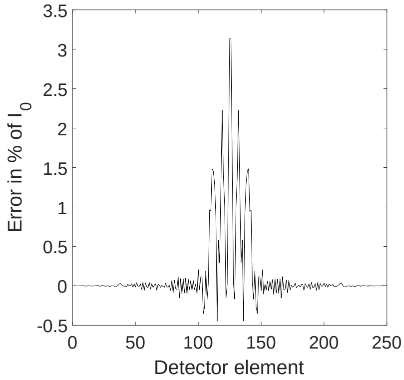


(a) Simulated sinogram with flat-field.



(b) Difference between ground truth and estimated flat-field.

Figure 1: Sinogram (a) and error in estimation (b).

III. Experiments and results

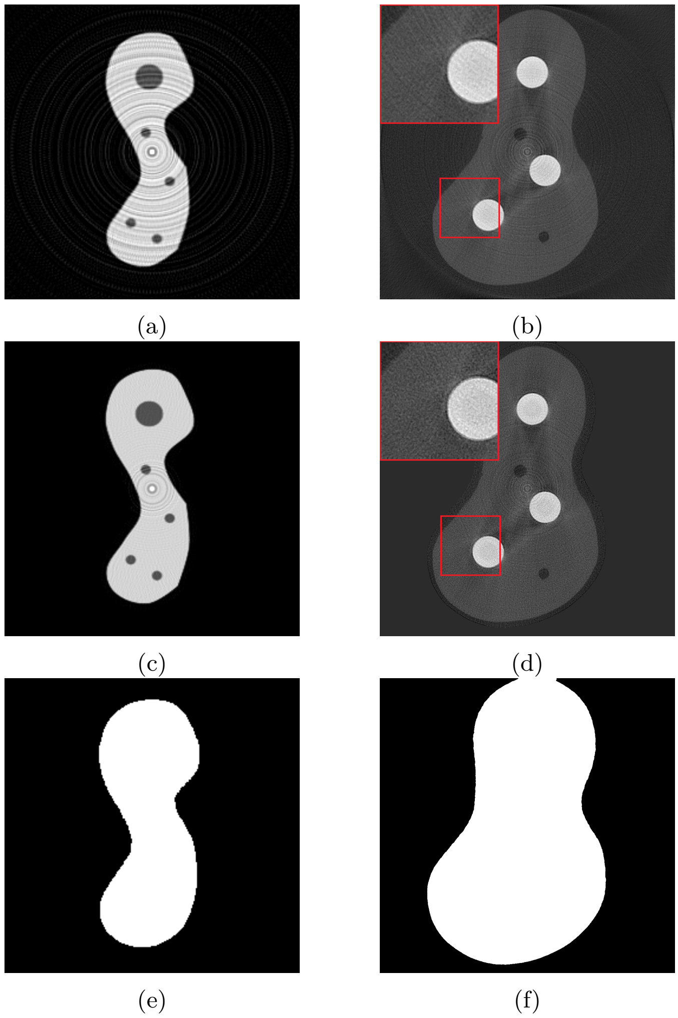
To validate our proposed method, we first reconstructed a simulated dataset with a randomly generated flat-field. We added a random flat-field to the parallel beam sinogram of a phantom image, as shown in Fig. 1a. Next, we reconstructed the central slice of an experimental dataset, for which no flat-fields were acquired. The imaged object for the experimental data was a plexiglass phantom with three aluminium rods inserted and two small drilled holes. The projection data was acquired using a SkyScan 1172 scanner on a 1000×1000 detector under 600 evenly spaced projection angles.

In both cases, a conventional reconstruction is computed with CG in 45 iterations, without estimating flat-fields. Next, we used the described method to calculate both the flat-field estimate and a reconstruction simultaneously. Here, 3 CG reconstructions were performed to obtain an estimate of the support, after which we used 45 CG iterations for the joint reconstruction and estimation. All forward and back projections using the system matrix were performed using the ASTRA toolbox [7].

In Fig. 1b the error between the ground truth flat-field and the estimated flat-field is plotted. This figure suggests that the estimation works well on most parts of the detector. In Fig. 2, we show the conventional and joint reconstructions of the object as well as the support used, for the simulated (left column) and real data (right column). In both real and experimental data one can observe a reduction of the ring artefacts, most noticeable for the rings which travel outside of the support at some point.

IV. Conclusion and future work

We presented a method that can simultaneously reconstruct an object and the flat-fields using linear solvers. Simulation and real experiments showed a significant reduction of ring artefacts in the reconstruction. The support estimation is at this moment calculated from an initial reconstruction, however, it would be interesting to include an iteratively changing support.



(a) (b) (c) (d) (e) (f)

Figure 2: Reconstructions with CG (a,b) and joint method (c,d), estimated support shown in (e,f).

Acknowledgements

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References

[1] F. E. Boas and D. Fleischmann, "CT artifacts: causes and reduction techniques," *Imaging Med*, vol. 4, no. 2, pp. 229–240, 2012.

[2] H. O. Aggrawal, M. S. Andersen *et al.*, "A convex reconstruction model for X-Ray tomographic imaging with uncertain flat-fields," *IEEE Transactions on Computational Imaging*, vol. 4, no. 1, pp. 17–31, 2018.

[3] E. X. Miqueles, J. Rinkel *et al.*, "Generalized titarenko's algorithm for ring artefacts reduction," *Journal of synchrotron radiation*, vol. 21, no. 6, pp. 1333–1346, 2014.

[4] J. Sijbers and A. Postnov, "Reduction of ring artefacts in high resolution micro-CT reconstructions," *Physics in Medicine & Biology*, vol. 49, no. 14, p. N247, 2004.

[5] P. Paleo and A. Mirone, "Ring artifacts correction in compressed sensing tomographic reconstruction," *Journal of synchrotron radiation*, vol. 22, no. 5, pp. 1268–1278, 2015.

[6] T. M. Buzug, *Computed tomography: from photon statistics to modern cone-beam CT.* Springer Science & Business Media, 2008.

[7] W. van Aarle, W. J. Palenstijn *et al.*, "Fast and flexible X-ray tomography using the ASTRA toolbox," *Optics Express*, vol. 24, no. 22, pp. 25 129–25 147, 2016.