Optimal estimation of T_2 maps from magnitude MR images

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ABSTRACT

A Maximum Likelihood estimation technique is proposed for optimal estimation of Magnetic Resonance (MR) T_2 maps from a set of magnitude MR images. Thereby, full use is made of the actual probability density function of the magnitude data, which is the Rician distribution. While equal in terms of precision, the proposed method is demonstrated to be superior in terms of accuracy compared to conventional relaxation parameter estimation techniques.

Keywords: T_2 map, Maximum Likelihood, magnitude MR images, Rice distribution

1. INTRODUCTION

Estimation of relaxation parameter maps has been a subject of considerable interest from the early years of Magnetic Resonance (MR) imaging. Along with the spin-lattice relaxation parameter T_1, the spin-spin-relaxation parameter T_2 gives useful information about the interaction with the local environment and plays a major role in the establishment of image contrast.

Conventional relaxation parameter estimation techniques applied to magnitude MR images are constructed from (weighted) least squares fitting procedures, which are only optimal in case of Gaussian distributed data.[1] Magnitude MR data however are Rician distributed. Recently, a paper was published on the use of the Rice distribution in the problem of estimating T_2 maps from magnitude MR data.[2] In that paper the problem on the data distribution was recognized but parameter estimation was still performed assuming Gaussian, additive noise. The authors justified the use of least squares estimation by stating that the Rician distribution approaches a Gaussian one at high signal-to-noise ratio (SNR). Although this is true, a bias is introduced in the estimation procedure which becomes more pronounced with decreasing SNR.

In this work, a Maximum Likelihood (ML) estimation technique is proposed for optimal estimation of the spin-spin relaxation times from a set of magnitude MR images. This choice can be justified because an ML estimator is known to be consistent and asymptotically most precise.[3] In the construction of the ML estimator, full use is made of the Rician distribution. The validity of the proposed method is checked by simulation experiments. Finally, the method is tested on experimental MR data.

2. METHOD

2.1. Magnitude data PDF

The real and imaginary components, $\{R_i\}$ and $\{I_i\}$, respectively, of the complex MR data are generally known to be corrupted by zero mean Gaussian noise.[4] The noise standard deviation (SD), which will be denoted by σ, can be estimated independently from homogeneous regions or from a double acquisition.[5,6]

Magnitude data $\{M_i\}$ are computed according to:

$$M_i = \sqrt{R_i^2 + I_i^2} \tag{1}$$

It is easy to show that the probability distribution (PD) of the magnitude data will be Rician,[7] given by:

$$p_M(M|f(\rho, T_2)) = \frac{M}{\sigma^2} \exp\left(-\frac{M^2 + f^2(\rho, T_2)}{2\sigma^2}\right) I_0\left(\frac{f(\rho, T_2)M}{\sigma^2}\right) \tag{2}$$

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where I_0 is the 0^{th} order modified Bessel function of the first kind. M denotes the pixel value of the magnitude image. $f(\rho, \mathrm{T}_2)$ is a function determined by the MR imaging sequence applied. In case the transversal magnetization decay is mono-exponential and conventional spin-echo imaging is performed, the following model is known to be accurate:

$$f_i(\rho, T_2) = \rho \exp\left(-\frac{TE_i}{T_2}\right) \tag{3}$$

with ρ denoting the pseudo-proton density which is a function of the true proton density and the T_1 parameter. The shape of the Rice distribution is strongly dependent of the signal-to-noise ratio (SNR), where the SNR is defined as the ratio $f(\rho, \mathrm{T}_2)/\sigma$. Fig. 1 shows the Rician PDF for various values of the SNR. From that figure one can observe that at high SNR, i.e. SNR > 3, the Rician PDF becomes quasi Gaussian. At low SNR the Rician PDF starts to deviate from a Gaussian one and finally becomes a Rayleigh PDF at SNR $= 0$. It is therefore expected that whenever parameter estimation techniques that were originally developed for Gaussian distributed data are applied to magnitude data, systematic errors will be introduced due to the asymmetry of the Rice PDF, especially at low SNR.

2.2. Errors introduced in T_2 estimation

In case of very high SNR the expectation of the relaxation behavior is given by Eq. (3) because at high SNR the value $f(\rho, \mathrm{T}_2)$ equals the expectation value of the (approximately Gaussian distributed) magnitude data. In general however, the expectation value of the magnitude data is given by:

$$E[M] = \sigma\sqrt{\frac{\pi}{2}} e^{-\frac{f(\rho,T_2)^2}{4\sigma^2}} \left[\left(1 + \frac{f(\rho,T_2)^2}{2\sigma^2}\right) I_0\left(\frac{f(\rho,T_2)^2}{4\sigma^2}\right) + \frac{f(\rho,T_2)^2}{2\sigma^2} I_1\left(\frac{f(\rho,T_2)^2}{4\sigma^2}\right)\right] \tag{4}$$

The deviation from $f(\rho, \mathrm{T}_2)$ becomes more pronounced with decreasing SNR. In Fig. (2) the expectation value $E[M]$ for T_2 relaxation is shown for various levels of the SNR. The true time constant was $\mathrm{T}_2 = 100$ ms and 100 for the pseudo proton density ρ.

2.3. Maximum Likelihood estimation

In this section the ML approach is clarified for the estimation of the unknown parameters set ρ and T_2 from a set of N independent magnitude data points $\{M_i\}$. The proposed technique consists of maximizing for each pixel position the joint probability density function (PDF), also referred to as the likelihood function, of N Rician distributed data points with respect to $\{\rho, \mathrm{T}_2\}$. The likelihood function of N independent magnitude data points is given by:

$$L(\{M_i\}|\rho, T_2) = \prod_{i=1}^{N} p(M_i|\rho, T_2) \tag{5}$$

$$\stackrel{\text{Eq.(2)}}{=} \frac{1}{\sigma^{2N}} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(M_i^2 + f_i^2(\rho, T_2)\right)\right) \prod_{i=1}^{N} M_i I_0\left(\frac{f_i(\rho,T_2)M_i}{\sigma^2}\right) \tag{6}$$

Maximization of L is equivalent to maximizing $\log L$ as log is a monotonic increasing function:

$$\log(L) = -N\log\sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(M_i^2 + f_i(\rho,T_2)^2\right) + \sum_{i=1}^{N}\log I_0\left(\frac{f_i(\rho,T_2)M_i}{\sigma^2}\right) + \sum_{i=1}^{N}\log M_i \tag{7}$$

For maximization of $\log L$, only the terms which are a function of the unknown parameters to be estimated are relevant:

$$\log(L) \sim \sum_{i=1}^{N}\left[\log I_0\left(\frac{f_i(\rho,T_2)M_i}{\sigma^2}\right) - \frac{f_i(\rho,T_2)^2}{2\sigma^2}\right] \tag{8}$$

Then the ML estimate for the parameter vector ρ and T_2 is the global maximum of $\log(L)$:

$$\left\{\hat{\rho}, \hat{\mathrm{T}}_2\right\}_{\mathrm{ML}} = \arg\left\{\max_{\rho,\mathrm{T}_2}(\log L)\right\} \tag{9}$$

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At high SNR, i.e., when the Rice distribution can be well approximated by a Gaussian PDF, the likelihood function becomes:

$$L(\{M_i\}|\rho, T_2) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{N}{2}} \prod_{i=1}^{N} \exp\left(-\frac{(M_i - f_i(\rho, T_2))^2}{2\sigma^2}\right) \tag{10}$$

In that case it is well known that maximization of $\log L$ with respect to the unknown parameters is equivalent to minimizing the quadratic distance E given by:

$$E = \sum_{i=1}^{N} [M_i - f_i(\rho, T_2)]^2 \tag{11}$$

This is also generally known as least squares (LS) fitting.

3. EXPERIMENTS AND DISCUSSION

To validate the proposed estimation method, experiments were set up using simulated as well as experimental MR data. All data processing tasks were performed on a Hewlett Packard 720 workstation.

3.1. Simulation experiments

To show that a bias is introduced in the estimation whenever Gaussian instead of Rician distributed data are assumed, a simulation experiment was set up. Thereby, real valued data, exponentially decaying according to Eq. (3), were corrupted with Gaussian distributed noise. Zero mean imaginary data were also polluted with Gaussian noise with the same SD. Magnitude data were then computed according to Eq. (1). From 16 Rician distributed data points, obtained in this way, ρ and T_2 were estimated, once using the conventional least squares (LS) fitting procedure and once using the proposed ML estimation technique. The estimation was repeated 10^5 times for each value of the SNR, which is defined as:

$$\mathrm{SNR} = \frac{\langle f(\rho, T_2)\rangle}{\sigma} \tag{12}$$

with $\langle f(\rho, \mathrm{T}_2)\rangle$ the average signal value:

$$\langle f(\rho, T_2)\rangle = \frac{1}{N}\sum_{i=1}^{N} f_i(\rho, T_2) \tag{13}$$

where $f_i(\rho, \mathrm{T}_2)$ is given by Eq. (3).

Fig. 3a and 3b show the results for the estimation of ρ and T_2, respectively. The true value for the pseudo proton density was $\rho = 100$, and 100 ms for the T_2 relaxation constant. Each time the average value was plotted as a function of the SNR. For clarity, the 95% confidence intervals are omitted: the relative error was of the order of 0.1% for both estimators. Both figures clearly demonstrate that the proposed ML technique is more accurate compared to conventional LS estimation. In case of high SNR, opposed to the outcomes of the LS estimator, no bias can be observed for the ML estimator. However, at low SNR (SNR < 5) the ML estimator can be seen to become biased though the bias is still significantly smaller compared to that obtained by LS estimation.

The shape of the likelihood function is shown in Fig. 4. It was observed that the two-dimensional $\log(L)$ function has only one maximum, corresponding to the ML estimate of ρ and T_2. The general shape of the likelihood function did not change for different values of the true ρ and T_2 parameters, nor for various SNR. As a result, because of the occurrence of only one maximum of the likelihood function, optimization becomes a very simple task: it can be performed using standard optimization techniques with no risk of getting stuck into a local maximum. Each ML estimate was obtained by maximization of the likelihood function using the downhill simplex method of Nelder and Mead in two dimensions.[8]

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3.2. T_2-map estimation

Next to the simulation experiment, tests were performed on experimental MR data. All data were generated on an MR apparatus (SMIS, Surrey, England) with a horizontal bore of 8 cm, a field strength of 7 Tesla and a maximal gradient strength of 0.1 Tesla/m. T_2 maps of a mouse brain were constructed in the context of studying the abnormalities in the ventricular system of hydrocephalic mice. The T_2 parameter is very sensitive to changes in water status as they occur in development and in response to pathology. Among others, T_2 depends on the ratio of the free-to-bound water in tissue. Changes in this ratio often occur in degeneration processes. E.g., T_2 increase due to the formation of vasogenic edema following an stroke event.
To acquire T_2-weighted images a 2D spin echo pulse sequence was used (TR = 1500 ms) with $FOV_x = FOV_y = 20$ mm and slice thickness 1 mm. The acquisition matrix 256 × 128 was zero filled to obtain a 256 × 256 image. From 6 magnitude images with echo times $TE_i = 20, 30, 40, 50, 60$ and 80 ms respectively the T_2 decay constant and the pseudo proton density was estimated for each pixel position. Two T_2-maps were obtained, one using the proposed ML estimation technique (shown in Fig. 5a) and one using LS estimation. Also the (intensity scaled) difference image was computed. This is shown in Fig.5b.

Finally we remark that in this paper a mono-exponentially decaying model was fitted to MR magnitude data points as to illustrate the consequences of not exploiting the proper data PDF. Naturally, as to make the imaging model more realistic, the model can be made arbitrarily complex by taking into account additional parameters. In that case a higher dimensional likelihood function needs to be maximized.

4. CONCLUSIONS

The use of the ML estimator is highly recommended as, compared to conventional estimators, the results are in general superior with respect to accuracy. Finally, as the likelihood function was observed to yield only one maximum, the computational requirements for the maximization are very low.

REFERENCES

1. J. Liu, A. Nieminen, and J. L. Koenig, "Calculation of T_1, T_2 and proton spin density in nuclear magnetic resonance imaging," *Journal of Magnetic Resonance* **85**, pp. 95–110, 1989.
2. J. M. Bonny, M. Zanca, J. Y. Boire, and A. Veyre, "T_2 maximum likelihood estimation from multiple spin-echo magnitude images," *Magnetic Resonance in Medicine* **36**, pp. 287–293, 1996.
3. A. Van den Bos, *Handbook of Measurement Science*, vol. 1, edited by P.H. Sydenham, (Wiley, Chichester, England), 1982. Chap. 8.
4. E. R. McVeigh, R. M. Henkelman, and M. J. Bronskil, "Noise and filtration in magnetic resonance imaging," *Medical Physics* **12**(5), pp. 586–591, 1985.
5. J. Sijbers, A. J. den Dekker, D. Van Dyck, and E. Raman, "Estimation of signal and noise from Rician distributed data.," in *Proceedings of the International Conference on Signal Processing and Communications*, (Gran Canaria, Canary Islands, Spain), February 1998.
6. J. Sijbers, A. J. den Dekker, M. Verhoye, J. Van Audekerke, and D. Van Dyck, "Estimation of noise from magnitude MR images," *Magnetic Resonance Imaging* **16**(1), pp. 87–90, 1998.
7. A. Papoulis, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, Tokyo, Japan, 2^{nd} ed., 1984.
8. W. H. Press, W. T. Vettering, S. A. Teukolsky, and B. P. Flannery, *Numerical Recipes in C*, University Press, Cambridge, 1994. Chap. 10.

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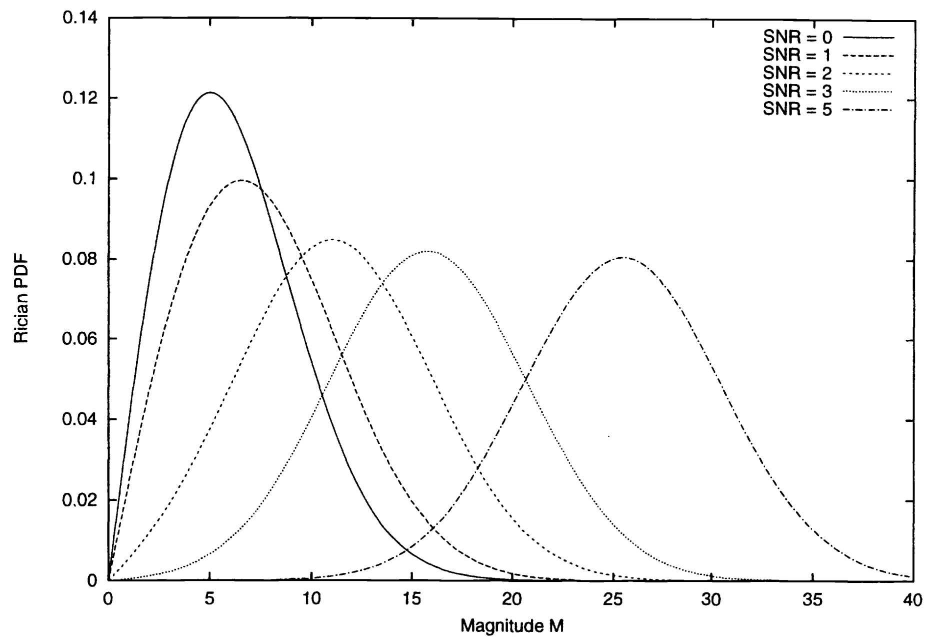

Figure 1. The Rician PDF as a function of the SNR.

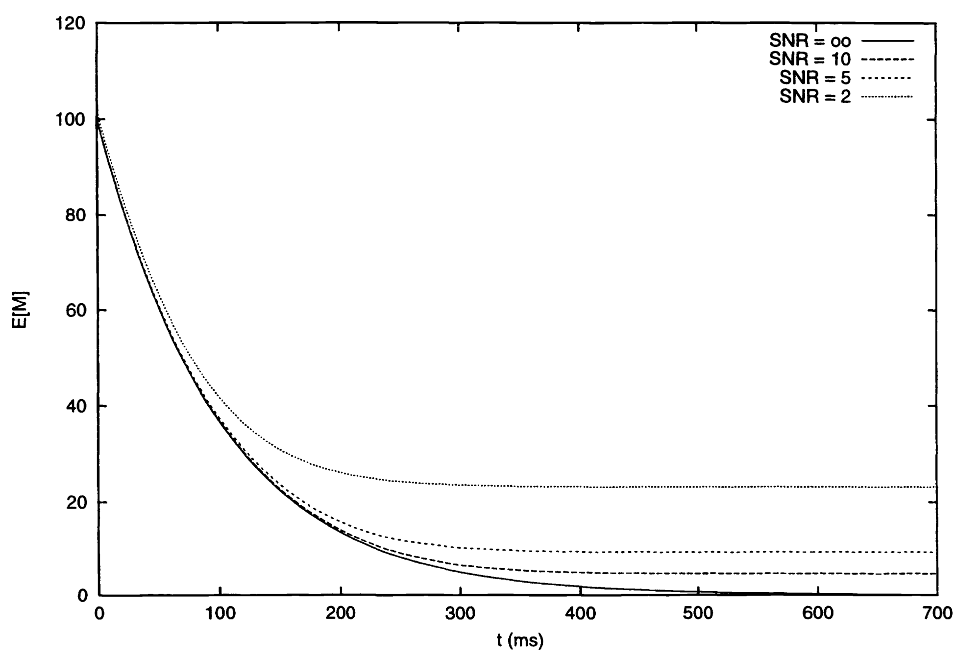

Figure 2. Expectation values of magnitude MR signal for T_2 relaxation as a function of the SNR. True values are $\rho = 100$ and $T_2 = 100$ ms.

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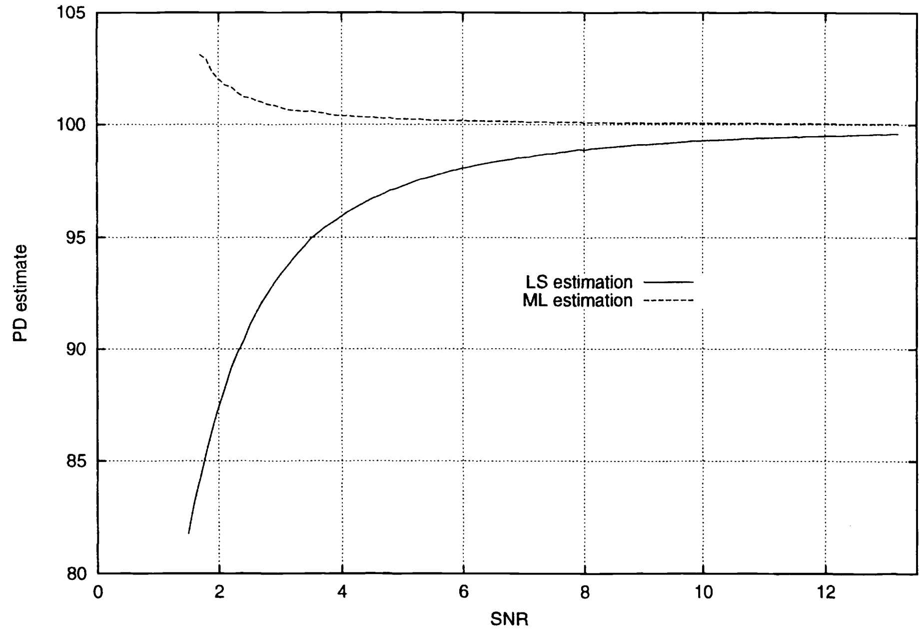
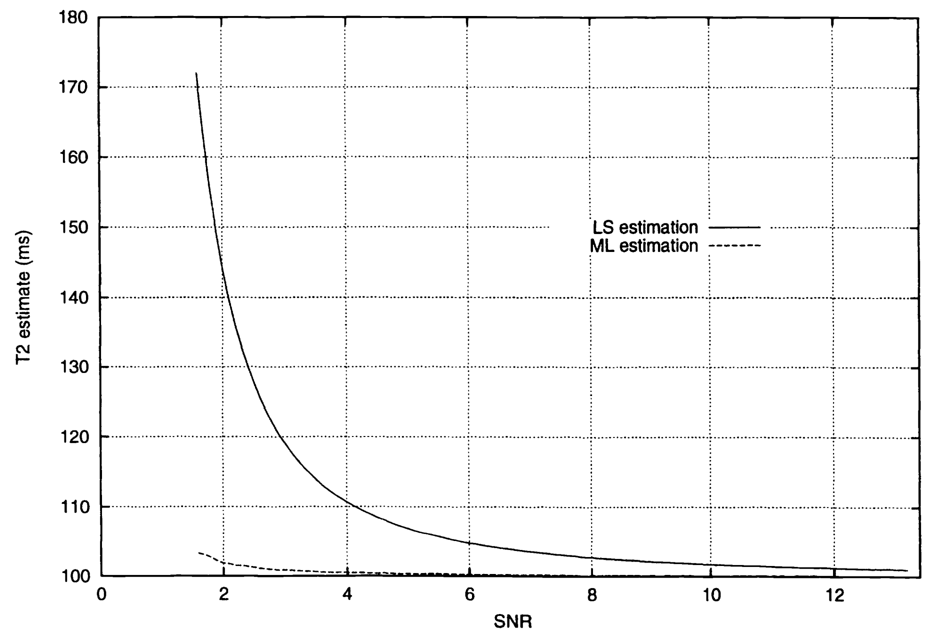


(a) PD estimation



(b) T_2 estimation

Figure 3. Simluation experiment: simultaneous ρ and T_2 estimation as a function of the SNR. True values are $\rho = 100$ and $T_2 = 100$ ms.

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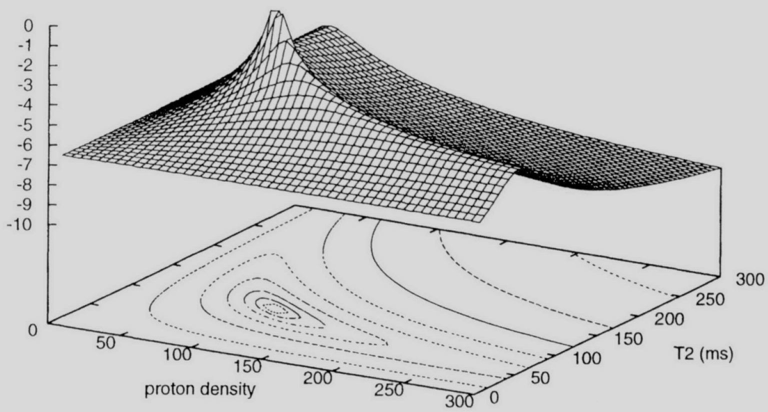

Figure 4. Shape of the $\log L$ function as a function of the pseudo proton density and T_2.

(a) T_2 map using ML

(b) Bias correction

Figure 5. T_2 maps of the mouse brain.

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