Reconstruction with rigid motion correction technique in CT imaging: A simulation and application study

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What is CT?

Figure 1: a CT scanner (source: Johns Hopkins Medicine).

Radon transform

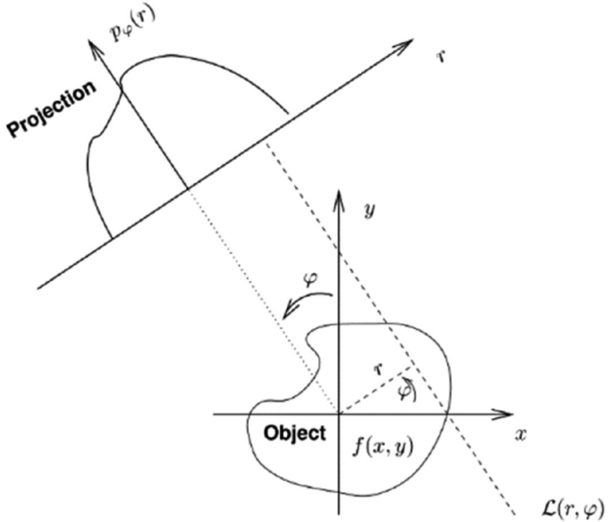

Figure 2: an interpretation of the Radon transform used in CT imaging.

A unique sample

Figure 3: the 7-carat diamond in front of the X-ray source of the CT scanner.

CT acquisition

Click!

Clip: angular-range X-ray projections of the diamond.

Motivation

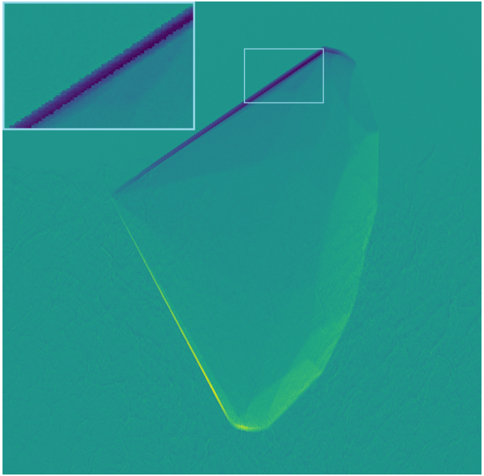

Figure 4: difference between the first and the last projections of the diamond.

Preliminaries

Moving object

A sequence of n images, $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$, each representing the moving object at a given time.

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Mathematical model of subscans

$$\boldsymbol{W}_i \boldsymbol{x}_i = \boldsymbol{b}_i, \text{ for } i = 1, \ldots, n,$$

where $\boldsymbol{W}_i$ and $\boldsymbol{b}_i$ are respectively the projection matrix and the projection data according to the i^{th} sub-scan.

Preliminaries (cont.)

Forward model

$$\begin{bmatrix} \boldsymbol{W}_1 & 0 & 0 & 0 \\ 0 & \boldsymbol{W}_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \boldsymbol{W}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_n \end{bmatrix}.$$

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Motion model

$$\boldsymbol{x}_i = M(\boldsymbol{p}_i)\,\boldsymbol{x},$$

where $M(\boldsymbol{p}_i)$ is the motion operator, parameterized by $\boldsymbol{p}_i$.

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Deformation vector field (DVF)

$$M(\boldsymbol{p}_i) \approx \mathrm{DVF}\left[\boldsymbol{x} \to \boldsymbol{x}_i\right].$$

Dynamic process model

Forward model with motion operator

$$\begin{bmatrix} \boldsymbol{W}_1 & 0 & 0 & 0 \\ 0 & \boldsymbol{W}_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \boldsymbol{W}_n \end{bmatrix} \begin{bmatrix} M(\boldsymbol{p}_1) \\ M(\boldsymbol{p}_2) \\ \vdots \\ M(\boldsymbol{p}_n) \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_n \end{bmatrix}.$$

More concisely,

$$\boldsymbol{W}\boldsymbol{M}(\boldsymbol{p})\boldsymbol{x} = \boldsymbol{b}.$$

The operator $\boldsymbol{W}\boldsymbol{M}(\boldsymbol{p})$ expresses the relation between the single static image $\boldsymbol{x}$ and the projection data $\boldsymbol{b}$ of the entire dynamic scan.

Motion-corrected reconstruction

Optimization approach [1, 2]

$$[\hat{\boldsymbol{x}}, \hat{\boldsymbol{p}}] = \operatorname{argmin}_{\boldsymbol{x},\boldsymbol{p}} f(\boldsymbol{x}, \boldsymbol{p}),$$

where

$$f(\boldsymbol{x}, \boldsymbol{p}) = \frac{1}{2} \left\| \boldsymbol{W} \boldsymbol{M}(\boldsymbol{p}) \boldsymbol{x} - \boldsymbol{b} \right\|_2^2.$$

[1] G. Van Eyndhoven et al., *Proceeding of the ECCV,* pp. 12-21, (2012).
[2] M. Zehni et al., *IEEE Trans. Image Process.,* **29,** pp. 6151–6163, (2020).

Gradient method

Iterative schemes

Initial guess: $\boldsymbol{p}^0 \equiv \boldsymbol{0}, \boldsymbol{x}^0 = \text{argmin}_{\boldsymbol{x}} f\left(\boldsymbol{x}, \boldsymbol{p}^0\right) \equiv \text{argmin}_{\boldsymbol{x}} \left\|\boldsymbol{W}\boldsymbol{x} - \boldsymbol{b}\right\|_2^2.$

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - \mu_{\boldsymbol{x}}^k \nabla_{\boldsymbol{x}} f\left(\boldsymbol{x}^k, \boldsymbol{p}^k\right),$$

$$\boldsymbol{p}^{k+1} = \boldsymbol{p}^k - \mu_{\boldsymbol{p}}^k \nabla_{\boldsymbol{p}} f\left(\boldsymbol{x}^k, \boldsymbol{p}^k\right).$$

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Stepsize quantization [1]

$$\mu_{\square}^k = \frac{\left\langle \nabla_{\square} f\left(\boldsymbol{x}^k, \boldsymbol{p}^k\right) - \nabla_{\square} f\left(\boldsymbol{x}^{k-1}, \boldsymbol{p}^{k-1}\right), \square^k - \square^{k-1} \right\rangle}{\left\|\nabla_{\square} f\left(\boldsymbol{x}^k, \boldsymbol{p}^k\right) - \nabla_{\square} f\left(\boldsymbol{x}^{k-1}, \boldsymbol{p}^{k-1}\right)\right\|^2}, \text{ where } \square = \boldsymbol{x} \text{ or } \boldsymbol{p}.$$

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[1] J. Barzilai and J. Borwein, *IMA J. Numer. Anal.*, **8**, (1), pp. 141–148, (1988).

Gradient method (cont.)

Gradient

The gradient of the objective function is given by $\nabla f = \left[\left[\nabla_{\boldsymbol{x}} f \right]^T, \left[\nabla_{\boldsymbol{p}} f \right]^T \right]^T$, where

$$\nabla_{\boldsymbol{x}} f\left(\boldsymbol{x}, \boldsymbol{p}\right) = \boldsymbol{M}\left(\boldsymbol{p}\right)^T \boldsymbol{W}^T \boldsymbol{r},$$

$$\nabla_{\boldsymbol{p}} f\left(\boldsymbol{x}, \boldsymbol{p}\right) = \left[\nabla \boldsymbol{M}\left(\boldsymbol{p}\right) \boldsymbol{x}\right]^T \boldsymbol{W}^T \boldsymbol{r},$$

with $\boldsymbol{r} = \boldsymbol{W}\boldsymbol{M}\left(\boldsymbol{p}\right)\boldsymbol{x} - \boldsymbol{b}$ is the residue of the system. Here, $\boldsymbol{W}$ is provided by the ASTRA Toolbox [1]; $\boldsymbol{M}$, $\boldsymbol{M}^T$ and $\nabla \boldsymbol{M}$ are provided by the ImWIP [2].

[1] W. van Aarle et al., *Ultramicroscopy*, vol. 157, pp. 35-47, (2015).
[2] J. Renders et al., *SoftwareX*, vol. 24, p. 101524, (2023).

Diamond real dataset

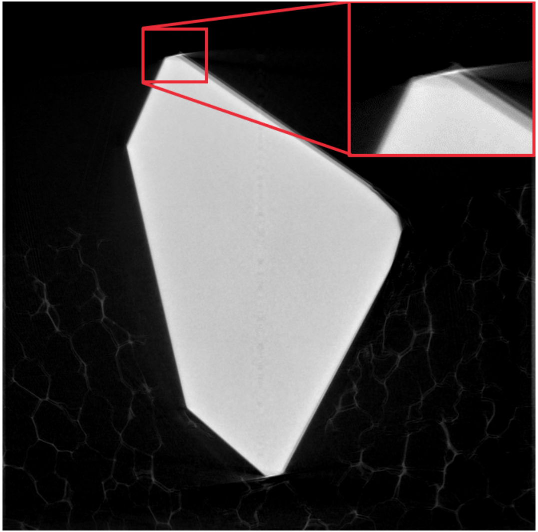
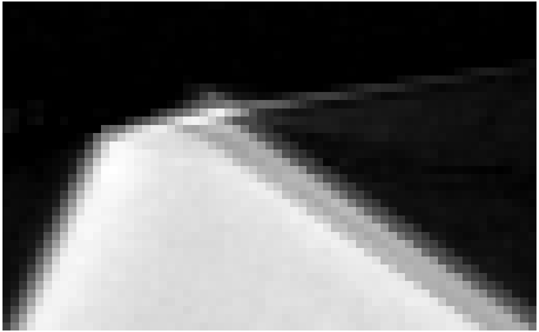
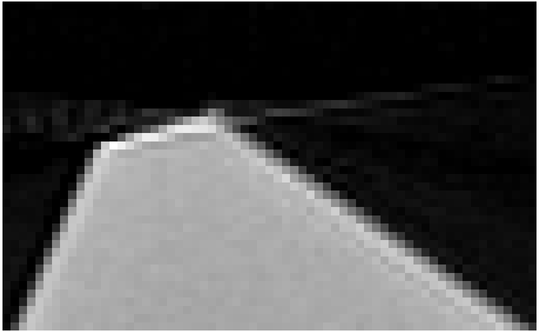


Figure 5: reconstruction without rigid motion compensation of the diamond.

Diamond real dataset



(a) without rigid motion correction



(b) with rigid motion correction

Figure 6: reconstruction results on the diamond's real projection dataset (volume size $472 \times 480 \times 480$ (voxel), voxel size 8 μm, 90 iterations, 1 min./iteration).

Diamond real dataset (cont.)

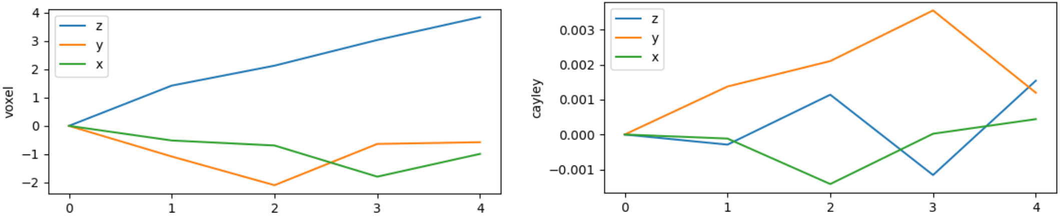


Figure 7: the estimated translations (left) and rotations (right).

Diamond simulated dataset

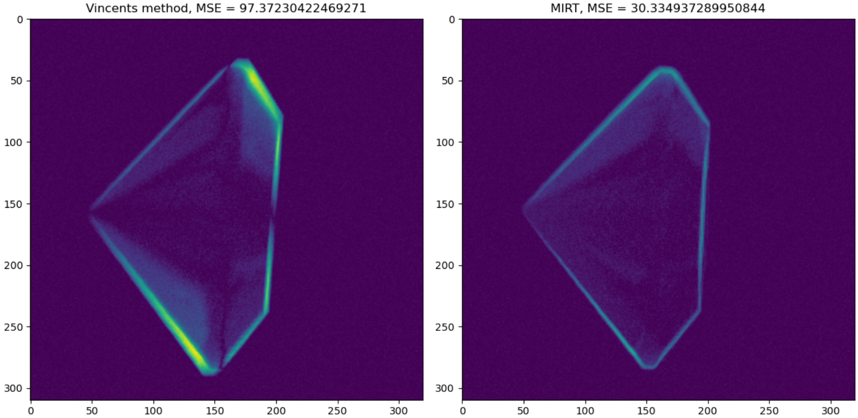
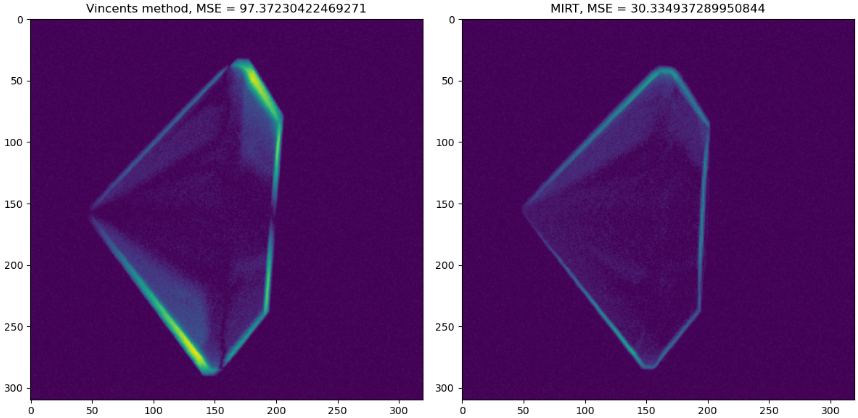


Figure 8: a comparison with [1], when [2] couldn't handle the volume.

[1] V. Van Nieuwenhove et al., IEEE Trans. Image Process., **26**, (3), pp. 1441-1451, (2017).

Diamond simulated dataset



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Conclusion and future work

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An iterative gradient-based dynamic 4DCT method that:

i) allows simultaneously accurate reconstruction and rigid motion parameter estimation.

ii) uses exact gradients and adjoints.

iii) does not contain nested iterations.

iv) outperforms relevant rigid-motion compensated CT reconstruction techniques in projection distance and computational feasibility.

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iv) outperforms relevant rigid-motion compensated CT reconstruction techniques in projection distance and computational feasibility.

Future work

i) acceleration using multiple GPUs for continuous motion estimation.
ii) feasible subscan partition problem.

Future work (cont.)

Problem (feasible subscan partition)

$$\left[\widehat{n}, \widehat{\sigma}_1, \ldots, \widehat{\sigma}_{\widehat{n}}\right] = \operatorname*{arg\,min}_{n, \sigma_1, \ldots, \sigma_n} g\left(n, \sigma_1, \ldots, \sigma_n\right),$$

with

$$g\left(n, \sigma_1, \ldots, \sigma_n\right) = n + \lambda \sum_{k=1}^{n} \sigma_k^2,$$

where $\sigma_k^2 = Var\left(\{s_l | l \in \mathcal{S}_k\}\right) \in [0, 1]$, and $\lambda > 0$ is the trade-off coefficient between the integer term and the statistical term.

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Theorem

Solution(s) to the problem “feasible subscan partition” exist only when the trade-off coefficient $\lambda > 1$.

THE END.

THANK YOU FOR YOUR ATTENTION!